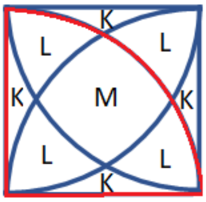
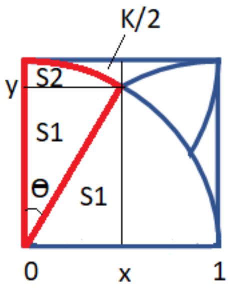
Solution to Math Skills Problem 4:

Rachid:



Totalareal:	$4K + 4L + M = 1$	(1)
Rødt areal:	$2K + 3L + M = \pi/4$	(2)
4x(2)- 3x(1):	$M = \pi - 3 + 4K$	(3)



$x = 1/2$; $x^2+y^2=1$

$y = \text{sqrt}(3)/2$

$S1 = x*y/2 = \text{sqrt}(3)/8$

$\text{Sin}(\Theta) = 1/2$

$\Theta = \pi/6$

Rødt areal: $S1+S2 = 1^2 * \Theta/2 = \pi/12$

$K/2 = (1/2*1) - (S1+S2) - S1 = 1/2 - \pi/12 - \text{sqrt}(3)/8$

$K = 1 - \pi/6 - \text{sqrt}(3)/4$ (4)

(3) og (4): $\underline{M = 1 + \pi/3 - \text{sqrt}(3)}$

George

Each circle is represented as an equation where (x + a)^2 + (y + b)^2 = 1. Placing the centre of the square at (0,0), then (a,b) belong to the complete set (0,0), (1,0), (1,1), (0,1), and there must be four crossing points @ (0.5,A),(A,0.5),(0.5,1-A),(1-A,0.5). To determine A:

sqrt(1-0.25) = y = sqrt(3)/2,

We can then do a double integral, but easier to visualise if you calculate the area under the circle (1) centred @ (0,0) between [0.5,sqrt(3)/2]:
S sqrt(1-x^2) dx (solve by substitution: x=sin(u)):
(x*sqrt(1-x^2) + arcsin(x))/2 | [1/2,sqrt(3)/2]
=>(sqrt(3)/4 + pi/3 - sqrt(3)/4 - pi/6)/2
=>(pi/6)/2

Isolating only that quadrant requires us to subtract the remaining area underneath, bound by the line y=0.5

S0.5dx | [1/2,sqrt(3)/2]
0.5x | [1/2,sqrt(3)/2]
=>(sqrt(3)-1)/4

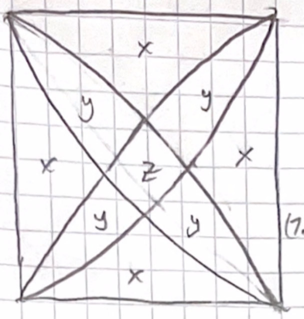
So, each shaded quadrant has an area of:
=>(pi/6)/2 - (sqrt(3)-1)/4

And thus the total shaded area is...
pi/3 - sqrt(3) + 1

Not able to typeset in LaTeX or anything right now. I feel like there should be a simple geometric solution almost immediately… the circle has a radius = 1. Each quadrant has an area of 0.25*pi. Two opposing arcs form an eye-shape with an area of the square - 2 * the external area left by one arc (which is 1 - 0.25*pi). So our 'eye' has an area of 0.5 * pi - 1. But I couldn’t then make the leap to remove the curved triangles without integrating, at which point it was simpler to do the above.

Håkon

$r=1$



We have

(1) $2x+y = 1-\pi/4$

(2) $2y+z = 1-2\cdot(2x+y) = \frac{\pi}{2}-1$

(1)·2:

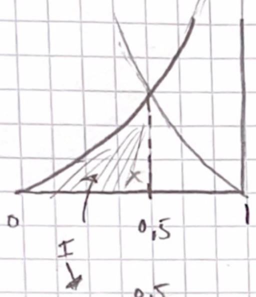
$\Rightarrow 4x+2y = 2-\pi/2$

$4x + \pi/2 - 1 - z = 2 - \pi/2$

$\Rightarrow z = \pi - 3 + 4x$

Look closer at x:

$r=1$



Simply a circle given by

$y = -\sqrt{1-x^2}+1$

$x = 2\cdot \underbrace{\int_0^{0,5} (-\sqrt{1-x^2}+1)\,dx}_{I}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$I = \int_0^{0,5} -\sqrt{1-x^2}\,dx + \int_0^{0,5} dx = \frac{1}{2} - \underbrace{\int_0^{0,5} \sqrt{1-x^2}\,dx}_{I_2}$, $x = \sin u$, $dx = \cos u\,du$

$\Rightarrow I_2 = \int (1-\sin^2 u)^{1/2} \cos u\,du$

$= \int \cos^2 u\,du = \frac{1}{2}\int 1+\cos 2u\,du = \frac{1}{2}\left(u + \frac{1}{2}\sin 2u + C\right)$

$\Rightarrow \frac{1}{2}\left[\sin^{-1}x + \frac{1}{2}\sin(2\cdot\sin^{-1}x)\right]_0^{1/2} = \frac{1}{2}\,\pi/6 + \frac{1}{4}\underbrace{\sin\left(\frac{\pi}{3}\right)}_{=\sqrt{3}/2}$

$\Rightarrow I_2 = \frac{\pi}{12} + \sqrt{3}/8$

$x = 2\cdot I = 2\cdot\left(\frac{1}{2} - \frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = 1 - \frac{\pi}{6} - \frac{\sqrt{3}}{4}$

which gives z:

$z = \pi - 3 + 4\cdot x = \pi - 3 + 4 - \frac{2\pi}{3} - \sqrt{3}$

$z = 1 - \sqrt{3} + \frac{\pi}{3} \approx 0,315$