

# IFT-Posten: Math Puzzle 7 Solution (week 46, 2022)

Olav Kalvbein's solution

The diagram shows a right-angled triangle  $\triangle EDB$  with the right angle at  $E$ . The vertical leg  $ED$  has a length of 5, and the horizontal leg  $EB$  has a length of 12. The hypotenuse is  $DB$ , which has a length of 13. A circle with center  $C$  and radius  $R$  is inscribed in the triangle, tangent to the legs  $ED$  and  $EB$  at points  $F$  and  $A$  respectively. A smaller circle with center  $G$  and radius  $\frac{1}{2}$  is tangent to the leg  $ED$  at point  $S$  and to the larger circle at point  $I$ . The distance from  $E$  to  $S$  is 5. The distance from  $A$  to  $B$  is 12. The distance from  $F$  to  $C$  is  $l$ . The distance from  $C$  to  $D$  is  $R$ . The distance from  $E$  to  $D$  is 5. The distance from  $E$  to  $B$  is 12. The distance from  $E$  to  $D$  is 5. The distance from  $E$  to  $D$  is 5.

Pythagoras gir hypotenus  $\sqrt{12^2 + 5^2} = 13$ . Fra  $\triangle GCF$  har vi

(\*)  $l^2 = \overline{FC}^2 = (R + \frac{1}{2})^2 - (R - \frac{1}{2})^2 = 2R$ .

Vitfra  $\triangle FCE$  og  $\triangle CDE$  er

$$\overline{EC}^2 = l^2 + (5 - R)^2 \quad \text{og}$$

(\*\*)  $\overline{ED}^2 = \overline{EC}^2 - R^2 = l^2 + 25 - 10R = 25 - 4l^2$ .

$\triangle ABC \cong \triangle CBD$  siden de har to like lange sider og en  $90^\circ$ -vinkel. Dermed er

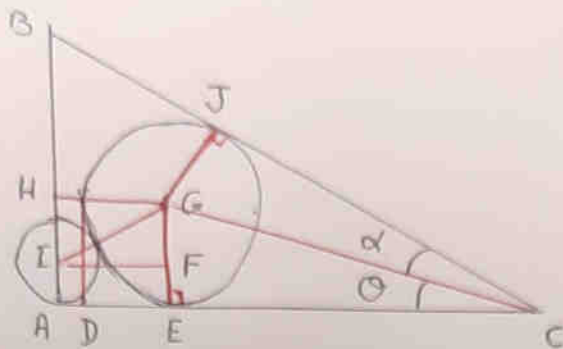
$$\overline{AB} = \overline{BD}$$
$$12 - l = 13 - \overline{ED}$$
$$\overline{ED} = 1 + l.$$

Vitfra dette og (\*\*\*) er

$$(1 + l)^2 = 25 - 4l^2$$
$$\frac{5}{2}l^2 + l - 12 = 0$$
$$l = \frac{-1 + \sqrt{1 + 4 \cdot \frac{5}{2} \cdot 12}}{2 \cdot \frac{5}{2}} = 2.$$

(\*) gir da  $R = 2$

Rachid Maad's solution



$$AI = 0,5 = EF; \quad AB = 5; \quad AC = 12$$

$$AD = x$$

$$EG = r; \quad IF = AE = x + r; \quad FG = r - 0,5; \quad IG = r + 0,5$$

$$\text{Triangel (IFG)}: (r+x)^2 + (r-0,5)^2 = (r+0,5)^2$$

$$\Rightarrow (r+x)^2 = 2r \Rightarrow \boxed{r+x = \sqrt{2r}}$$

$$\text{Det er lett å se at: } \alpha = \theta; \quad BC = \sqrt{12^2 + 5^2} = 13$$

$$\tan \theta = \frac{1 - \cos(2\theta)}{\sin(2\theta)} = \frac{1 - 12/13}{5/13} = \frac{1}{5}$$

$$\tan \theta = \frac{r}{EC} = \frac{r}{12 - (r+x)} = \frac{r}{12 - \sqrt{2r}}$$

$$\Rightarrow \frac{1}{5} = \frac{r}{12 - \sqrt{2r}} \Rightarrow 5r = 12 - \sqrt{2r}$$

$$a = \sqrt{r} \Rightarrow 5a^2 + \sqrt{2}a - 12 = 0$$

$$\Rightarrow a = \sqrt{2} \Rightarrow \boxed{r = 2}$$

$$\underline{\underline{\text{Radius} = 2}}$$

( $r+x = \sqrt{2r} \Rightarrow x = 0 \Rightarrow AB$  er tangent til den store sirkelen)