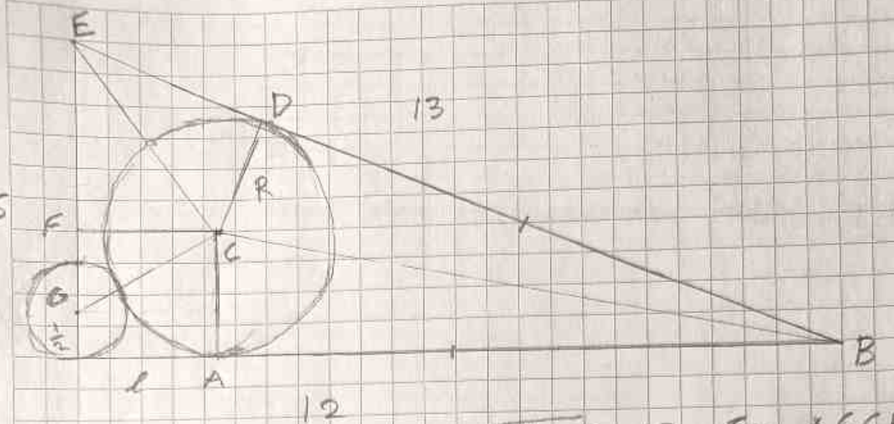
IFT-Posten: Math Puzzle 7 Solution (week 46, 2022)

Olav Kalvbein's solution



Pytagoras gir hypotenus $\sqrt{12^2+5^2}=13$. Fra $\triangle GCF$ har vi

(*) $\ell^2 = \overline{FC}^2 = (R+\frac{1}{2})^2 - (R-\frac{1}{2})^2 = 2R$.

Utifra $\triangle FCE$ og $\triangle CDE$ er

$\overline{EC}^2 = \ell^2 + (5-R)^2$ og

(**) $\overline{ED}^2 = \overline{EC}^2 - R^2 = \ell^2 + 25 - 10R = 25 - 4\ell^2$.

$\triangle ABC \cong \triangle CBD$ siden de har to like lange sider og en 90°-vinkel. Dermed er

$\overline{AB} = \overline{BD}$

$12 - \ell = 13 - \overline{ED}$

$\overline{ED} = 1 + \ell$.

Utifra dette og (**) er

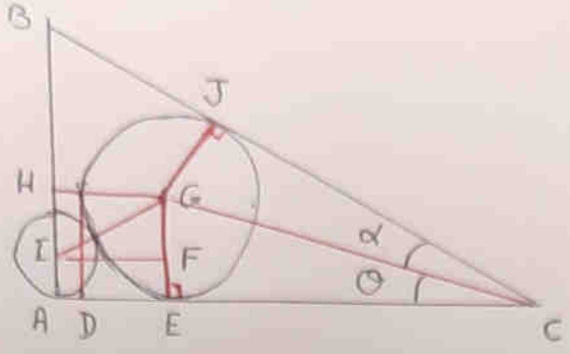
$(1+\ell)^2 = 25 - 4\ell^2$

$\frac{5}{2}\ell^2 + \ell - 12 = 0$

$\ell = \dfrac{-1+\sqrt{1+4\cdot\frac{5}{2}\cdot 12}}{2\cdot\frac{5}{2}} = 2.$

(*) gir da $\underline{R = 2}$

Rachid Maad’s solution



$AI = 0,5 = EF$; $AB = 5$; $AC = 12$

$AD = x$

$EG = r$; $IF = AE = x + r$, $FG = r - 0,5$, $IG = r + 0,5$

Triangel (IFG): $(r+x)^2 + (r-0,5)^2 = (r+0,5)^2$

$$\Rightarrow (r+x)^2 = 2r \Rightarrow \boxed{r + x = \sqrt{2r}}$$

Det er lett å se at: $\alpha = \theta$; $BC = \sqrt{12^2 + 5^2} = 13$

$$\tan\theta = \frac{1 - \cos(2\theta)}{\sin(2\theta)} = \frac{1 - 12/13}{5/13} = \frac{1}{5}$$

$$\tan\theta = \frac{r}{EC} = \frac{r}{12 - (r+x)} = \frac{r}{12 - \sqrt{2r}}$$

$$\Rightarrow \frac{1}{5} = \frac{r}{12 - \sqrt{2r}} \Rightarrow 5r = 12 - \sqrt{2r}$$

$$a = \sqrt{r} \Rightarrow 5a^2 + \sqrt{2}\,a - 12 = 0$$

$$\Rightarrow a = \sqrt{2} \Rightarrow \boxed{r = 2}$$

<u>Radius = 2</u>

($r + x = \sqrt{2r} \Rightarrow x = 0 \Rightarrow AB$ er tangent til den store sirkelen)