List of Courses

This document will summarize the courses completed during my bachelor in Engineering Mathematics at *Universidad EAFIT*, Colombia (hereafter addressed as EAFIT).

Preliminaries

Credit Conversion to ECTS

According to article 11, paragraph 1 (page 20) of the academic regulation for bachelor programs (only available in Spanish) at EAFIT, one (1) academic credit is equivalent to forty eight (48) hours of academic work from the student.

Furthermore, according to the ECTS system, one (1) credit is equivalent to twenty five (25) to thirty (30) study hours in countries with the highest values. For this document, it will be assumed that one ECTS credit is equivalent to thirty study hours.

Hence, one (1) academic credit in Colombia is equivalent to 0.625 ECTS-credits. This factor was used to convert all course credits.

List of Courses

The course descriptions provided here were all written by me, based on my experience on my Bachelor in Engineering Mathematics at EAFIT, and the descriptions available at the university courses descriptions. All course descriptions can be found in this Drive folder, although not all of them are in English, this is why I wrote this document manually.

Modeling and Simulation I (4.8 ECTS)

Hours per semester: 48
Course code: CM0075
Software/language used: Excel, Simul8 and Matlab
Summary:
This course is the introductory course to the bachelor in Engineering Mathematics (EM). The following topics were covered:

- Engineering and science, the current role of engineering in the world, the objectives, curriculum, profile and possibility for graduate studies of the bachelor in EM, research possibilities at EAFIT, experiences from graduated students.
- Introduction to mathematical modeling, principles of modeling, use of mathematical models, classification of models, introduction to Excel and Matlab.

- Introduction to simulation, Monte Carlo simulation principles, introductions to discrete event simulation in Simul8, continuous simulation, and agent-based simulation.
- Introduction to optimization, linear programming and exploratory data analytics in Excel.

Fundamentals of Programming (6.4 ECTS)

Hours per semester: 64
Course code: ST0242
Software/language used: Java
Summary:
This course is the introduction to programming, from the Computer Science department. In general, the course covered the following topics:

- Introduction to the process of programming: actors, stages, functional and non-functional requirements, entities, assignations and arithmetic expressions.
- Algorithms: definition, properties, efficiency vs. effectiveness, lexic, syntaxis, semantics and decisions.
- Types, expressions and operators: Basic types, arithmetic and logical operators, ifs, loops, Java syntax.
- Arrays: looping patterns, arrays, syntaxis, types.
- Matrices: integers, characters, strings, Java libraries.
- Functions: parameters by value, parameters by reference, returning values.
- Object-Oriented Programming: definition, classes, objects, instances, methods, attributes and design patterns.

Calculus 1 (4.8 ECTS)

Hours per semester: 72
Course code: CM0230
Software/language used: N/A
Summary:
This course covers all differential calculus. In particular, the following topics were covered:

- Definition of function, linear functions, rational functions, plotting rational functions, domain and range of a function, functions of real variables.
- Limits and their properties: definition of limit, graphic analysis of limit, limit by analytical and numerical methods, the intuition of continuity, lateral and infinite limits, asymptotes.
- Differentiation: the derivative and the tangent line problem, basic differentiation rules, rates of change, the product and quotient rules, derivatives of higher order, the chain rule, and implicit differentiation.
- Application of differential calculus: extrema on intervals, Rolle's theorem and mean value theorem, increasing and decreasing functions, first and second order conditions for function analysis, limits to infinity, graph analysis, and optimization problems.

Logic (6.4 ECTS)

Hours per semester: 72

Course code: CM0260
Software/language used: LogicCoach
Summary:
This course is a shared course with the students in Computer Science. It is the introduction to logic theory and proof methods. In particular, this course covered the following topics:

- Semantics of propositional logic: symbolic representation of arguments, tautologies, contradictions and contingencies, truth tables, equivalence, contradiction, consistency and/or inconsistency between statements.
- Inference on propositional logic: inference rules, direct proofs, method of conditional proof, method of indirect proof, logical laws.
- First-order predicate logic: finite models, general propositions and unique propositions, quantifiers and inference rules for them.
- Formal logic: relations, symbols for relations, arguments involving relations, attributes of relations, identities and definite descriptions of relations, higher-order logic.
- Set operators: sets, union, intersections, difference, symmetric difference, complement, power sets, Venn diagrams.
- Proof by induction.

Geometry in Context (6.4 ECTS)

Hours per semester: 80
Course code: CM0446
Software/language used: N/A
Summary:
This course introduces the elementary axioms to construct geometry from a set theory perspective, never appealing to visual representation to justify statements, and some of the most important lemmas and theorems to prove with the given axioms. This course develops the logical-deductive abilities in the students and introduces them to the world of formal, strict and rigorous proofs in mathematics.

In particular, this course covered the following topics:

- Propositional logic
- Proof methods
- Basic set theory, equivalente and order relations
- Incidence axioms, collinearity, parallelism and Playfair's axiom
- Axioms of betweenness, segments, rays, angles and interior of an angle
- Axioms of congruence of segments, sum/difference/order of segments
- Axioms of congruence of angles, adjacent, supplementary and vertically opposite angles, addition/order of angles, right angles and Hilbert planes

Modeling and Simulation II (4.8 ECTS)

Hours per semester: 48
Course code: CM0078
Software/language used: Matlab

Summary:

This course is the continuation of the modeling and simulation courses in the bachelor. In particular, the following topics were covered:

- Modeling of deterministic problems: definition of model, suggestions for constructing models, examples of models in economics, engineering, physics, biology and their solution through geometry or elementary mathematics, exponential models in finance, biology, ecology, physics and chemistry, mathematical models using Linear Algebra, network models, introduction to graph theory, Euler and Hamitonian circuits, introduction to linear programming, feasible regions, convex sets and graphical solution to simple linear programming problems in two variables.
- Modeling of stochastic phenomena: random phenomena, sampling concepts, data descriptors, random variable and the concept of probability, least-squares linear regression, definition of some probability distributions (uniform, Poisson, Binomial, exponential, normal, gamma and Weibull), introduction to reliability and survival theory.
- Simulation and the Monte Carlo method: definitions and concepts, random number generation, Monte Carlo integration and solution to games using Monte Carlo.

Calculus 2 (4.8 ECTS)

Hours per semester: 72
Course code: CM0231
Software/language used: N/A
Summary:

This course covers the general topics of integral calculus. In particular, the following:

- Integration: definition of antiderivative, notion of area under the curve, definite integral, definition, properties and interpretation in terms of area, the fundamental theorem of calculus, and integration by substitution.
- Transcendental functions: definition, properties, derivative and integral of the natural logarithm, definition of inverse functions and injectiveness, relationship between the graph of a function and its inverse (whenever it exists), definition, properties, derivative and integral of the exponential and inverse trigonometric functions.
- Areas, methods of integration and improper integrals: area between curves, volume of a solid of revolution, integration by parts, trigonometric integrals, integrals containing powers of sine and cosine functions, trigonometric substitution, integration of rational functions, undetermined forms and the L'Hôpital rule, improper integrals and integrals with infinite limits.
- Sequences and series: definition and convergence of sequences, definition of series, partial sums and convergence of series, geometric series and divergence criterion, integral criterion, p-series, comparison of series, alternating series, absolute convergence and quotient criterion, representation of functions in power series, Taylor and Maclaurin series.

Linear Algebra (4.8 ECTS)

Hours per semester: 72

Course code: CM0234
Software/language used: N/A
Summary:
This course sets the base on Linear Algebra for the complete bachelor. This course covered the following topics:

- Matrices: definition and use cases, types of matrices, operations between matrices (sum and scalar product), the space vector of matrices and properties, dimension, multiplication of matrices.
- Linear equation systems: elementary row operations, the row-echelon form, range of a matrix, linear equation systems and matrix representations, augmented matrix, theorems on the solution of a linear equation system, transposed of a matrix, symmetric and antisymmetric matrices, homogeneous systems, inverse of a matrix, properties, solution through inverse matrix.
- Determinants: definition of determinant of a matrix, properties, calculation of determinants by cofactors, adjoint matrix, inverse matrix and Cramer’s rule.
- Vector spaces: definition and conditions that must satisfy, the space of real matrices, linear combinations, vector subspaces, subspace generated by a set, definition of a basis, definition of linear dependence and independence, relevant theorems, real vectors in 2D and 3D, magnitude and direction of vectors, unitary vectors, dot product, properties and applications, orthonormal sets and orthonormal bases, the Gram-Schmidt orthogonalization process, cross product and applications, lines and planes on the space, areas and volumes on the space.
- Eigenvalues and eigenvectors: definition and examples of eigenvalues and eigenvectors of a matrix, the characteristic polynomial of a matrix, similar matrices and matrix diagonalization, diagonalization of symmetric matrices, and applications.

Physics 1 (6.4 ECTS)

Hours per semester: 80
Course code: CM0230
Software/language used: N/A
Summary:
This course introduces the students to the basics of physics and mechanics. In addition to standard lectures, this course included several laboratory sessions, where the concepts were evidenced and/or applied to real phenomena. In particular, the following topics were covered:

- Introduction to physics: the nature of physics and the physics in nature, physics as an academic topic and the links to engineering.
- Units, physical magnitudes and measurements: unit systems, prefixes and orders of magnitude, introduction to vectors and vector algebra, measurement equipment and error-propagation theory.
- Kinematics: displacement and time, speed, velocity and acceleration, unidimensional motion, uniform straight line motion, non-uniform motion, free-fall motion, projectile kinematics.
- Newton laws and application: force and interactions, linear momentum and conservation, Newton first, second and third law, mass, weight and universal gravitational law,

applications of Newton laws, circular motion dynamics, friction and real-world engineering, equilibria of forces.
- Work and energy: kinetic work and energy, variable-force work, potential energy (gravitational and elastic), mechanical energy and principle of conservation, applications, energy diagrams and force, quantity of motion.
- Rotational dynamics: angular velocity and acceleration, link between angular and linear kinematics, rotational energy and moment of inertia, torque and angular acceleration of rigid bodies, angular momentum.
- Periodic motion: oscillations and harmonic motion, energy of harmonic motion, applications, vibrations and oscillations in engineering.
- Mechanical waves: types of mechanical waves, periodic waves, mathematical description of a wave, velocity of transverse waves, interference of waves, standing waves and normal modes.

Data Structures and Algorithms 1 (4.8 ECTS)

Hours per semester: 48
Course code: ST0245
Software/language used: python
Summary:
This course is the introduction to data structures and algorithms, algorithm complexity and selection of tools for specific computational tasks. Apart from the standard lectures, this course had several assignments and laboratories on each of the topics covered. In general, these topics were:
- Complexity of algorithms, the big O notation, definition, properties and arithmetics, examples.
- Lists, stacks and queues data structures, implementations and basic methods. Hash tables.
- Recursion, stop criterion, implementation and problem solving using recursion, algorithm complexity for recursion, recurrence relations and mathematical induction.
- Tree-based data structures, complexity of operations, advantages and disadvantages, implementation.
- Introduction to graphs, problems that can be modeled using graphs and graph-visit algorithms (breadth-first and depth-first search), shortest path algorithms, implementation.

Modeling and Simulation III (4.8 ECTS)

Hours per semester: 48
Course code: CM0079
Software/language used: Vensim
Summary:
This course is the introduction to system dynamics and systemic thinking. In particular, the course covered the following topics:

- Introduction to system thinking and system theory: systemic approach, general theory of systems, barriers to learning, causality, complexity and systemic thinking.
- Introduction to system dynamics: principles and process of modeling with system dynamics, causal diagrams, elementary structure and behavior of systems and applications.
- System dynamics simulations: levels and flows, growth dynamics, delays, co-flows, aging-chains and nonlinearities, modeling in Vensim.
- Output analysis: validation and verification of models, output analysis and application cases.

Calculus 3 (4.8 ECTS)

Hours per semester: 48
Course code: CM0415
Software/language used: Matlab
Summary:
This course covers topics in multivariable calculus, in particular:

- Vectors and functions of various variables: quadratic surfaces, functions of several variables, vectors in the plane and space, scalar (dot) and vector (cross) products, limits and continuity in several variables, partial derivatives, the multivariate chain rule, directional derivatives and gradients, normal vectors, tangent planes and optimization of multivariable functions (Lagrange multipliers).
- Multiple integrals: iterated integrals, double integrals and volume, change of variables, polar coordinates, center of mass and moments of inertia, triple integrals in cylindrical and spherical coordinates, applications.
- Vector fields, line integrals and surfaces: conservative vector fields and path independence, line integrals, surface integrals, Green's theorem, parametric surfaces, the divergence theorem and Stokes' theorem.

Discrete Mathematics (4.8 ECTS)

Hours per semester: 72
Course code: CM0246
Software/language used: N/A
Summary:
This course covered the following specific topics:

- Mathematical induction, strong induction, recursive definitions and structural induction for proofs over discrete sets.
- Set theory, basic definitions, properties of sets, counter-examples, algebraic proofs, boolean algebra and Russell's paradox.
- Functions and infinite sets: definition of function over any set, injective, surjective and bijective functions, inverse functions, function composition, domain, co-domain and range, set cardinality and applications to computability, infinite sets and discrete sets, the rational numbers as a discrete set (enumerations).

- Relations: definition of relation over any set, reflexivity, symmetry and transitivity, equivalence relations, partial and total order relations.
- Graphs: Definitions, types, basic properties, paths, routes, circuits, matrix representation and proof of statements on graph theory.

Physics 2 (4.8 ECTS)

Hours per semester: 80
Course code: DF0239
Software/language used: N/A
Summary:
This is the continuation of the physics courses. This course covered the following particular topics:

- Electric fields and potential: electric charge, Coulomb's law, electric field, electric force for point-charges, electrical field flux and Gauss' law, potential electrical energy, and electrical potential for point-charges, capacitance and dielectrics, energy on a capacitor, current, resistance and resistivity, electromotive force and electrical circuits.
- Magnetic fields: the magnetic field and sources, magnetic force, magnetic flux, motion of charged particles in fields, magnetic force induced by current flux, magnetic field of a charge in motion, Ampère's circuital law and applications, induction, Faraday's and Lenz's law, energy of a magnetic field, Hall effect, earth's magnetic field and Biot-Savart's law.
- Maxwell equations and electromagnetic waves: the Maxwell equations, electromagnetic waves, electromagnetic spectrum, flat electromagnetic waves, sinusoidal electromagnetic waves, electromagnetic waves on matter, refraction index, energy and momentum of electromagnetic waves.
- Optics: the nature of light, reflection and refraction of light, total internal reflection, Fresnel-Huygens' principle, interference and coherent sources, lasers, interference of light, diffraction, single-slit refraction, Young experiment, polarization of light, geometrical optics, Michelson and Twyman-Green interferometers, diffraction patterns, Fresnel and Fraunhofer approaches to diffraction, X-rays diffraction and Laue's experiment.

Probability Theory (4.8 ECTS)

Hours per semester: 48
Course code: CM0415
Software/language used: Matlab
Summary:
This course introduces the concept of probability from a formal point of view. This course covered the following topics:

- Induction, set theory, algebras, σ-algebras, generated σ-algebra, Borel's σ-algebra, and probability spaces.
- Kolmogorov axioms for a probability measure, properties of a probability measure.
- Conditional probability, Bayes theorem and law of total probability

- Random variables, distribution function, discrete and continuous random variables, mass and density functions, expected value and variance, moments of a random variable, skewness and kurtosis, moment-generating function and characteristic function.
- Some discrete distributions: Poisson, binomial, negative-binomial, hypergeometric. Some continuous distributions: normal, exponential, gamma, beta, Weibull, lognormal.
- Joint density and distribution functions of random variables, independence of random variables, covariance and Pearson correlation, random vectors and multivariate distributions introduction.

Formal Languages and Automata (4.8 ECTS)

Hours per semester: 48
Course code: CM0081
Software used: Haskell
Summary:
This course is an introduction to formal theory of computing while strengthening the knowledge in discrete mathematics and rigorous proofs.. This course covered the following topics:
- Deterministic and non-deterministic finite automata, without and with ϵ-transitions, for accepting languages.
- Introduction and algebra of regular expressions, properties of closure and the pumping lemma.
- The Turing machine, languages accepted by Turing machines, extensions and Turing-computable functions.
- The Church-Turing thesis, the universal Turing machine, undecidable problems and hypercomputation.

Modeling and Simulation IV (4.8 ECTS)

Hours per semester: 48
Course code: CM0080
Software/language used: Matlab & Simulink
Summary:
This course was the introduction to dynamical systems and their implementation in Simulink, as preparation for the Linear Systems and Experimental Modeling courses. In general, this course covered the following topics:
- Introduction and motivation to mathematical models, types and classification, definition of dynamic system, measurement errors, resolution, precision, accuracy, observation of dynamic systems in engineering, elementals of data pre-processing and filtering, outliers, model validation, Ockham's razor principle, model selection and parameter estimation, classification of optimization common optimization methods, metrics for goodness-of-fit (SSE, R^2), block-representation of dynamic systems and introduction to Simulink, different approaches to modeling (differential equations, integral representation, analytical solution), real-world dynamic systems, implementation and simulation.

- Modeling and simulation in the state space: state variables, state space, outputs vs. state variables, state equation, normalization of variables, equilibrium points, phase plane portraits, introduction to control, perturbations, time delays, strange attractors and chaos.
- Additional topics in dynamic systems: stability criterion for linear systems, non-linear dynamic systems, linearization process by Taylor series, Jacobian matrix, control of linear systems, feedback controllers for linear systems, PID control in Simulink, uncertainty and sensitivity analysis, sensitivity indices, parameter estimation in Simulink.

Differential Equations (4.8 ECTS)

Hours per semester: 72
Course code: CM0235
Software/language used: N/A
Summary:
This course is the formal introduction to ordinary differential equations (ODEs) and analytical methods of solution. In particular the following topics were addressed:

- First-order ODEs: fundamentals, verification of solutions, initial value problems, separation of variables, linear ODEs, special substitutions, exact ODEs, applications on mixing and population models, and Euler's method for numerical approximation.
- Higher-order ODEs: the mass-spring oscillator, linear differential operators, fundamental solutions of homogeneous equations, basics of complex analysis and Euler's formula, the superposition principle and non-homogeneous linear higher-order ODEs, undetermined coefficient methods, variation of parameters, the Cauchy-Euler equations, and applications on free and damped oscillators.
- The Laplace transform: definition and properties of the Laplace transform, basic transforms and inverse Laplace transform, solution to initial value problems using the Laplace transform, the Laplace transform of discontinuous and periodic functions, the convolution operator and its transform, the Laplace transform of integrals, integro-differential equations and solution to systems of coupled ODEs.

Statistics 1 (4.8 ECTS)

Hours per semester: 48
Course code: CM0418
Software/language used: R
Summary:
This course covers the topics related to univariate statistical inference, using the knowledge from the course in Probability Theory. In particular, the following topics were addressed:

- Transformation of random variables: the Student's t and F distribution.
- Sample distributions, central limit theorem, properties of point estimators, method of moments, method of maximum likelihood.
- Parametric confidence intervals with large and small samples for the mean, difference of means and variance.

- Hypothesis testing, large sample tests, estimating probability of false positives, size of test estimation, hypothesis concerning small samples and variance, test of independence, homogeneity and goodness-of-fit.
- The general linear statistical model, least-squares estimation and properties, determination coefficient and inference on parameter estimation.

Optimization 1 (4.8 ECTS)

Hours per semester: 48
Course code: CM0426
Software/language used: AMPL with Gurobi, and Matlab
Summary:
This course is the first approach to optimization and operations research in the program, giving the students solid foundations to learn and face new optimization problems in the future. In general, this course covered the following topics:

- Introduction to optimization problems, linear programming problems (LPP) with continuous, integer, binary and mixed variable spaces, and problem modeling and formulation.
- Simplex method for solving LPPs and its variants, elements of duality theory, sensitivity analysis on the solution, fundamentals of integer LPP solvers.
- Formulation, solution model and methods for optimization problems on networks: transportation problem, allocation problem, maximum flow problem, shortest route problem, minimum expansion tree problem.

Modeling and Simulation V (4.8 ECTS)

Hours per semester: 48
Course code: CM0085
Software/language used: Simul8, python and NetLogo
Summary:
This course is the culmination of the modeling and simulation courses, touching base on discrete and agent-based simulation. In particular, the following topics were covered:

- Discrete-event simulation (DES): fundamentals, terminology, context and application cases, conceptual modeling of DES models, random number generators and random variables, Poisson processes, queues, statistical analysis of input data (independence and homogeneity tests, goodness-of-fit tests), introduction to Simul8 and python's SimPy library for DES, output data analysis (biases elimination, estimation of appropriate number of runs, confidence intervals and sensitivity analysis), validation and verification of DES models, experiment design and optimization-simulation basics.
- Agent-based modeling and simulation (ABMS): fundamentals, terminology, context and application cases, conceptual modeling of ABMS models, introduction to NetLogo for implementing simple ABMS models, output data analysis (sensitivity analysis, parameter estimation and fitting, robustness and uncertainty analyses), validation and verification of ABMS models and experimenting in ABMS models.

Optimization 2 (4.8 ECTS)

Hours per semester: 48
Course code: CM0050
Software/language used: Matlab
Summary:
This course is the continuation of the optimization line, covering topics in nonlinear programming, multi-objective and multi-attribute optimization and a brief introduction to stochastic optimization. In particular, this course covered the following topics:

- Convex and concave multivariable functions, convex programming, Lagrangian formulation, Karush-Kuhn-Tucker optimality conditions, and numerical methods for unconstrained and constrained nonlinear optimization problems.
- Modeling of multi-objective problems, Pareto efficiency, relaxation to single-objective optimization, methods for solution.
- Classification of multi-attribute methods and representative problems. Methods of solution.
- Criteria of decision, programming with probabilistic constraints, random objective functions, stochastic programming with resources.

Partial Differential Equations (4.8 ECTS)

Hours per semester: 48
Course code: CM0422
Software/language used: N/A
Summary:
This course is the theoretical introduction to the analytical and approximate solutions of elementary partial differential equations (PDEs), identifying the appropriate method for each equation. This course covered the following particular topics:

- Definition of PDEs, classification of PDEs, superposition principle, separation of variables, generalized Fourier series, periodic functions, even/odd functions, piecewise continuous function, real and complex Fourier series in sines and cosines, convergence analysis and Fourier integral.
- Fourier transform, theorems, transformation of different functions and operators, and solution of PDEs.
- The wave equation, heat equation, Laplace equation and Poisson equation: formulation, special cases, solution approaches and transformations. The d'Alembert's formula for the wave equation.
- Dirichlet problems in cartesian and polar coordinates, special cases, and finite sine and cosine transforms.
- Neumann problem, Green’s first identity, necessary conditions for existence of the solution, and Neumann problem in polar coordinates.

Statistics 2 (4.8 ECTS)

Hours per semester: 48
Course code: CM0423
Software/language used: R
Summary:
This course is the continuation of the statistics line, now addressing multivariate statistics for data analytics. This course covered the following particular topics:

- Introduction to multivariate analysis: random vectors, mean vectors, covariance and correlation matrices, linear combination of variables, measures of total variability, the Mahalanobis distance.
- Inference and hypothesis testing on mean vectors: the multivariate normal distribution, Hotelling's T-squared distribution, the Wishart distribution, hypothesis testing on a mean vector with known and unknown covariance matrix, comparison of two mean vectors, paired observation testing, the discriminant function and linear discriminant analysis.
- Principal component analysis: Definition and properties of the main components, variance-optimization approach, geometric approach, relation with eigenvalues and eigenvectors of covariance matrix, main components of the correlation matrix, interpretation and dimensionality reduction.
- Multivariate linear regression: least-squares approach, inference on the parameters, and regularization.

Numerical Analysis (ECTS 4.8)

Hours per semester: 48
Course code: ST0256
Software/language used: python
Summary:
This course is shared with the computer science department. It is the introduction to numerical algorithms (and some theory) for the solution of common problems in applied mathematics and computer science. In particular, this course covered the following topics:

- Error theory, error propagation, precision and accuracy, stability of numerical algorithms, and representation of floating point numbers in computers.
- Nonlinear equation solvers, incremental and interval search methods, bisection, fixed-point, Newton-Raphson and secant iterative methods. Conditions for convergence.
- Numerical solution to linear equation systems: gaussian elimination, pivoting, LU matrix decomposition, Jacobi, Gauss-Seidel and SOR methods, error and convergence analysis, vector and matrix norms.
- Interpolation: Newton polynomial and Lagrange interpolators, linear, quadratic and cubic splines.
- Numerical differentiation and integration: trapezoid and Simpson’s rules for integration.
- Introduction to the numerical solution of ordinary differential equations: convergence, consistency and stability, Euler’s method, Taylor series-based methods and the Runge-Kutta methods.

Analysis 1 (4.8 ECTS)

Hours per semester: 48
Course code: CM0087
Software/language used: N/A
Summary:
This course was the introduction to real analysis. This course covered the following topics:

- Real number system: axioms of the real numbers, properties of order, density and completeness, and bounded sets.
- Topology of metric spaces: definition of a metric, open and closed sets, definition of closure, accumulation point and boundary of a set, and relevant theorems.
- Convergence and completeness: definition of sequence, convergence of sequences in metric spaces, properties of limits in metric spaces, characterization of a metric space by the limit of its sequences, monotonous sequences and convergence, Cauchy sequences, complete metric space, compactness, and connectedness.
- Continuity: formal definitions of continuity and equivalence between them, continuity in compact metric spaces, sequences of functions, and point and uniform convergence of sequences of functions.

Heuristics (4.8 ECTS)

Hours per semester: 48
Course code: CM0439
Software/language used: python
Summary:
This course is the culmination of the optimization courses in the bachelor, it introduces concepts and common heuristic methods for solving combinatorial optimization problems. In particular, this course covered the following topics:

- Introduction to heuristic methods: definition, computational complexity, P, NP and NP-hard problems, properties of heuristic methods, formulation of combinatorial optimization problems, and definition of lower and upper bounds for optimization problems (constraints relaxation).
- Classical search methods: depth-first, breadth first, bidirectional search, best-first, A*, hill-climbing, branch and bound.
- Constructive methods: definition, principles, advantages and disadvantages, implementation, lower and upper bounds, and strategies for evaluating the quality of proposed algorithms.
- Randomized constructions: definition, principles, advantages and disadvantages, randomization for heuristics, implementation, ant colony optimization, parameter tuning.
- Local search methods: definition, definition, principles, advantages and disadvantages, initial solutions, movements, neighborhood definition, local optima, strategies for escaping local optima, variable neighborhood search, search with memory (Tabu search), probabilistic acceptance (simulated annealing), multiple initial solutions (GRASP), perturbations (ILS), parameter tuning, and stop criteria.

- Evolutionary algorithms: definition, definition, principles, advantages and disadvantages, principles, population, selection, crossover, mutation, evolutionary strategies, genetic algorithms, particle swarm optimization, differential evolution, parameter tuning, and stop criteria.

Stochastic Processes 1 (4.8 ECTS)

Hours per semester: 48
Course code: CM0433
Software/language used: Matlab and python
Summary:
This course is the introduction to stochastic processes, probabilistic models and queue theory. In particular, the following topics were covered:

- Definition of stochastic process, Markov chains, transition matrix, classification of states, limit distributions, probability of a stable state, average first pass times estimation, and absorbent chains.
- Poisson process/birth and death processes: counting processes, the Poisson process, Poisson and exponential distribution, distribution of inter-arrival and standby times, conditional distribution for the time between arrivals, pure birth process, pure death process, birth and death process, renewal process, and inhomogeneous Poisson process.
- Queue processes: Kendall-Lee notation for queueing processes, wait times and expected cost optimization, multiple particular cases of queue processes, infinite source models, models in series and open queuing networks.
- Martingales: conditional expectation, martingales in discrete time, definitions, properties and relevant theorems.

Linear Systems (4.8 ECTS)

Hours per semester: 48
Course code: CM0440
Software/language used: Matlab with Simulink
Summary:
This course aims to apply the concepts and methods of dynamic systems in discrete and continuous time for analysis and control of nonlinear systems. In particular, this course covered the following topics:

- Mathematical modeling of linear systems: general dynamic systems in continuous and discrete time governed by ordinary differential equations (ODEs), state space, nonlinear ordinary differential equations, special cases and solutions, concepts of stability (eigenvalues) and state diagrams (Simulink), solution to the continuous linear state equation, state transition matrix, matrix exponential and multivariable Taylor series, linear transforms and canonical forms, discretization of continuous systems with and without delays, solution of the discrete linear state equation, Laplace transform, properties, solution to ODEs using the Laplace transform, the Dirac delta, the Z transform, properties, basic transforms and inverse transform, solution of difference equations using the Z transform, transfer functions in continuous and discrete time for SISO systems, poles and zeros, systems with delays, order reduction and pole elimination, discretization of continuous transfer functions, signal sampling, matrix of transfer functions, zeros and poles of MIMO models, signal-flow graph and Mason formula.

- Analysis of dynamical systems: definition of Lyapunov stability, direct and indirect Lyapunov methods for stability, Routh-Hurwitz method for continuous linear systems stability, Jury method for discrete linear systems stability, root locus, frequency analysis, Bode diagrams for frequency properties and stability, signal sampling, aliasing and filtering.
- Control of linear systems: the problem of control and design requirements, stationary state error suppression, PID controllers, interpretation, discretization of PID controllers, saturation, pole assignment method by state feedback, separation principle, controllability and observability of linear systems and Ackermann's method.

Experimental Modeling (4.8 ECTS)

Hours per semester: 48
Course code: CM0089
Software/language used: python
Summary:
This course is the introduction to the theory of system identification for dynamical systems, hence, the following topics were generally addressed, as some of them are beyond the scope of undergraduate courses. In particular, this course covered the following topics:

- Linear regression, analytical solution to multivariate linear regression by convex quadratic programming, matrix conditioning, ill-posed linear regression, convex functionals
- Introduction general linear models (GLMs): R, AR, MA, ARMA, ARMAX, ARIMA, ARIMAX and their representation on GLM standards, the lag operator, lag covariances, L-polynomial and stationarity of GLMs (Hurwitz criterion), FIR systems.
- Parameter identification, definition of asymptotically consistent estimators, optimal estimators, observability of state and the state-observer linear model, regularization, generalized derivatives (Gateaux, Fréchet, directional and Dini derivatives), analytical solution to ARMAX least-squares parameter estimation, biased estimators, recursive least-squares variant, the Sherman-Morrison formula, properties of the recursive least-squares estimator.
- Theory of identifiability, problems, differential algebraic equations, q-identifiable systems, partially identifiable systems, the Cayley-Hamilton theorem, linearization of systems into GLMs.
- Characterization of stochastic processes, colored noises, introduction to the statistical part of the Kalman filter, Gaussian processes, state-observer covariances, state estimation by Kalman filter in discrete time, innovation-correction algorithm for filtering, instrumental variables for eliminating correlations, the Luenberger observer, the Kalman filter for parameter estimation.
- Frequency analysis, transfer function and frequency response function estimation based on real measurements, Levy optimization and minimization of error.

- Stability analysis of the Kalman filter: introduction to Lyapunov stability theory, asymptotic stability, Lyapunov functions and positive-definite functions, Lyapunov's theorem for stability of dynamic systems, Lie derivatives, application of Lyapunov's stability theory to the Kalman filter variance, Ricatti equations.

Analysis 2 (4.8 ECTS)

Hours per semester: 48
Course code: CM0088
Software/language used: python
Summary:
This course is the continuation of the analysis in the bachelor, moving into measure theory and some functional analysis. This course covered the following specific topics:

- Measurable sets and measures: σ-algebras revisited, definition of a measure on an a set, measurable sets, examples of measures, the Lebesgue measure in real numbers, properties and relevant theorems, the Borel and Lebesgue σ-algebras, examples of non Lebesgue-measurable set in the real numbers: Vitali sets.
- Measurable functions and the integral: definition of measurable functions and properties, the concept of "almost everywhere", characterization of measurable function, the analogy to continuity, the Riemann integral, Lebesgue integrable functions, the Lebesgue integral, concepts of advanced theory of probability, probability as a measure, theorems and examples, and the Radon-Nikodym theorem.
- Banach spaces: vector spaces, definition of norm, completeness and Banach space definition, p-integrable and bounded function spaces, modes of convergence and extension to probability theory.
- Topics of functional analysis in control theory: Affine control systems, approximability, uniformly Lipschitz continuous functions, uniformly bounded sets, Carathéodory functions and existence theorem, absolutely continuous functions and weak convergence of functions.

Stochastic Processes 2 (4.8 ECTS)

Hours per semester: 48
Course code: CM0438
Software/language used: python
Summary:
This course is the culmination of the stochastics line of knowledge in the bachelor. This course covers the following topics:

- Brownian motion: characteristic function and Gaussian processes, definition and properties of the Brownian motion, processes derived from the Brownian motion (drift, bridge and geometrical), continuous martingales, martingales associated to the brownian motion, Paley-Wiener and Levy representation, and simulation of brownian motions.
- Stochastic integrals: Paley-Wiener-Zygmund stochastic integral, definition and properties of the Itô integral, the Itô integral as a martingale, and extensions.

- Itô’s formula: one-dimensional Itô’s formula, multi-dimensional Itô’s formula, simulation of an Itô process, computational implementation of Itô’s formula, and the martingale representation theorem.
- Stochastic differential equations (EDEs): verification of solutions of systems of EDEs, existence and uniqueness theorem, the EDE as a mathematical model, analytical methods of solution and explicit solutions, numerical approximation methods (Euler-Maruyama and Milstein), simulation of EDEs, and solution of coupled systems.
- EDEs as financial models: actions and commodities, stochastic models of interest rates, natural gas price and electric power, identification and parameter estimation for some EDEs.
- The Black-Scholes model: Model assumptions, formulation and explicit equation proof, free-risk probability, Girsanov’s theorem, option pricing formulas, out-call parity, buy and sale options.
- Numerical methods for options assessment: Monte Carlo simulation, binomial trees, explicit finite differences.

Artificial Intelligence (4.8 ECTS)

Hours per semester: 48
Course code: CM0091
Software/language used: Julia and python
Summary:
This course integrates the knowledge developed throughout the bachelor in data science, in the context of artificial intelligence and machine learning. In particular, this course covered the following topics:

- Fuzzy logic: fuzzy sets and systems, fuzzy logic from a probabilistic approach, fuzzy logic algebra, membership functions, T and S norms, fuzzification methods, fuzzy inference systems and fuzzy controllers.
- Unsupervised learning: theory and implementation from scratch of k-means, fuzzy-c-means, spectral clustering, subtractive clustering and mountain clustering, embeddings and t-SNE.
- Supervised learning: activation functions, analytical solution to perceptron problem, theory and implementation of artificial neural networks from scratch, regularization, momentum, learning rate, cost functions and stochastic gradient descent.
- Introduction to deep learning: basics of convolutional neural networks, autoencoders for dimensionality reduction, ANFIS, GAN and StyleGAN.

Numerical Analysis 2 (4.8 ECTS)

Hours per semester: 48
Course code: CM0092
Software/language used: FEniCS API for python
Summary:

This course is the introduction to the functional analysis theory, and implementation of finite differences and finite element methods for approximating the solution of ordinary and partial differential equations. In particular, this course covered the following topics:

- Banach and Hilbert spaces: norm, normed spaces, vector spaces, convergence in normed spaces and Banach spaces, inner product, spaces with inner product, Hilbert spaces, orthogonality and direct sum.
- Linear operators: definition, theorems, null space and image space of a linear operator, isomorphisms and isometries, lower bounded operators, inverse operators, fixed-point theorem, linear functional and bilinear forms, Riesz representation theorem, dual spaces.
- Distribution theory and Sobolev spaces: locally integrable functions, test function space, definition of distribution, examples, regular distributions, weak derivatives, the Dirac delta, Sobolev spaces and Sobolev's immersion theorem.
- Finite element method (FEM): Dirichlet problem and Green identities, variational (weak) formulation of elliptic problems with Dirichlet and Neumann conditions, the Lax-Milgram theorem, Galerkin and Petrov-Galerkin methods, weighted residues method, finite elements, bidimensional problems and triangulation, FEM for diffusion and error estimation.

Research Practice 1, 2 and 3

Hours per semester: N/A
Course code: CM0436, CM0441 and CM0055
Software/language used: python
Summary:
These courses combined are the equivalent of the bachelor's thesis, they are included in the mandatory subjects in the last three academic semesters. On each one of them, the student must choose a research project and a supervisor to work with; although the specific topics are not limited, they must be inside the research areas of the Mathematical Sciences Department and EAFIT. During each course, the student (with the supervisor) must propose a research topic of interest and present a project proposal stating the problem definition, justification, methodology, state-of-the-art and estimated schedule during the semester. Once the project is approved, the research is conducted and evaluated with a mid-term presentation, a final paper-like report, and a final presentation with posters (all in English) to all the academic community in the Department.
In my particular case, I did all my three research practices with former professor Vadim Azhmyakov, in topics related with optimal control theory and hybrid systems. A more thoroughly explanation will be presented:

- **Research practice 1:** *"A Formal Approach to Optimal Strategic Decisions in Security Policies"* where we formalized a known optimal control problem for terrorist organization control based on a ODE-driven dynamical system, with a better formulation of the mathematical elements involved in the dynamic optimization problem. The problem was then solved and validated using a gradient-based numerical approach, implemented in python.
- **Research practice 2:** *"On the Approximate Optimal Feedback Control for ODE Involved Dynamic Systems"* where we addressed the famous optimal feedback control problem

by proposing an approximation based on β-relaxations, control-affine systems and functional analysis.

- **Research practice 3:** *"On the Geometrical Optimality Conditions for Optimal Control of Hybrid Systems"* where we explored the geometric interpretation of Sussman maximum principle for hybrid systems-involved optimal control, taking inspiration from the well-developed geometric theory for standard optimal control problems.

Unfortunately, due to professional reasons surrounding my supervisor and the university, we were not able to pursue a publication of any of these projects so far.

Mathematical Economics (4.8 ECTS)

Hours per semester: 48
Course code: EC0311
Software/language used: N/A
Summary:
This course summarizes the theory and applications of multivariable calculus, optimization and control theory in the context of economic models. The course covered the following particular topics:

- Continuity of real functions, ε-δ definition, calculus definition, relevant theorems from differential calculus, differentiability and differentials, convex sets, convex functions, directional derivatives, gradient, (semi) positive-definite matrices, Jacobians, implicit function theorem.
- Optimization from a calculus perspective, Hessian matrix, necessary and sufficient optimality conditions, optimization with constraints, Lagrange multipliers, bordered Hessian matrix, Karush-Kuhn-Tucker conditions for optimality.
- Ordinary differential equations (ODEs): first and second order ODEs, special cases and analytical solution methods, solution of systems of ODEs through linear algebra, phase plane and state space, linearly independent sets, Wronskian and linear ODEs of superior order, superposition theorem, first and second order discrete time difference equations, systems of difference equations and solutions.
- Introduction to optimal control: dynamic optimization fundamentals, calculus of variations, the Euler-Lagrange equations, Beltrami identity, the brachistochrone problem, Hamiltonians, transversality conditions, state and costate equations, analytical solution to elementary optimal control problems.

Nonparametric Statistics (4.8 ECTS)

Hours per semester: 36
Course code: CM0896
Software/language used: Matlab and python
Summary:
(This course is part of the Master in Applied Mathematics at EAFIT) This course is an introduction to nonparametric and robust statistical methods for data science. In particular, the following topics were covered:

- Robust and nonparametric version of classical descriptive statistics with properties, multivariate outlier detection, ordered statistics and distributions, distribution estimation and plug-in estimators, nonparametric and robust covariances and correlations (Kendall, Spearman, MAD, comedian), influence function, Bootstrapping and Jackknife with nonparametric estimation of confidence intervals.
- Smoothers and linear smoothers, linear and logistic regression, smoothing parameter and cross-validation, penalized regression, splines, LOWESS, Kernel density estimation, Nadaraya-Watson regression, robust linear regression, and regularized methods.
- Range-based statistical tests for sample inference (Wilcoxon and Mann & Whitney tests), statistical depth functions (Mahalanobis and Tukey), depth-based statistical tests, sample reduction using depth and robust methods.

Differential geometry (4.8 ECTS)

Hours per semester: 32
Course code: CM0887
Software/language used: N/A
Summary:
(This course is part of the Master in Applied Mathematics at EAFIT) This course is an introduction to differential geometry, focusing on constructing the basic concepts and theorems for starting in this field. In particular, this course covered the following topics:

- Visual introduction to 2D and 3D smooth surfaces, with boundaries, self-intersecting, with corners.
- Open and closed sets, bounded sets and compactness, intrinsic geometries in 3D surfaces, open questions.
- Towards the definition of regular surface: topological spaces, subspaces of a topological space, relevant theorems, definition of continuity from topology, homeomorphisms, functions of class C^r, infinitely-differentiable functions and smooth functions, diffeomorphic functions and sets, differential of a function on a given point, vector and Euclidean spaces, neighborhoods, equivalence relation of diffeomorphisms.
- K-manifold definition, coordinate system on manifolds, parameterizations, coordinate function, regular surface.
- Directional derivative, linear operators, the chain rule in open sets, the best linear approximation, tangent spaces, parallel spaces, tangent vectors, the generalized chain rule in manifolds, the inverse function theorem, the local immersion theorem and the embedding theorem.

Information Sciences (4.8 ECTS)

Hours per semester: 48
Course code: ST0291
Software/language used: N/A
Summary:
(Elective course) This course is an introduction and overview of relevant topics in information theory. In general, this course covered the following topics:

- The information revolution and language of information, general definition of information, life-cycle of information, definition of data, analogical vs. digital data, ethics of information, types of information: ambience, semantic, facts, physical, biological, economical.
- Shannon's mathematical information theory, syntax vs. semantic, the principle of information conservation, notions of quantum computing, information in biology and neurosciences, the genetic code, and information as a contemporary asset.
- Communication models, redundancy and noise, entropy, content of information and randomness, inference, information codification and notions of cryptography.
- Data comprehension, probability and inference, entropy, correlation and autocorrelations, pattern recognition and learning algorithms, geometry of information, information processing and algorithms.
- Introduction to complex dynamical systems, Conway's game of life, chaos, determinism, non determinism and predictability, butterfly effect and sensitivity to initial conditions.

Time Series (4.8 ECTS)

Hours per semester: 48
Course code: EC0301
Software/language used: python and R
Summary:
(Elective course) This course covered the following topics:

- Introduction to forecasting in economics and introduction to stochastic processes.
- Linear models: autoregressive (AR), moving average (MA) models, mixed models (ARMA) and integrated models (ARIMA), autocorrelations and partial autocorrelations, exogenous variable models (ARIMAX), the lag operator and stationarity, stationarity tests (Dickey-Fuller, Phillips-Perron and KPSS), invertibility, random walks.
- Construction of an ARIMA model, identification, estimation, validation and forecasting, the Yule-Walker equations, conditional estimation of ARIMA models (OLS, gradient-iterative and MLE), non-conditional estimation, exact likelihood estimation and stational models (SARIMA), white noise validation (Box-Pierce and Ljung-Box tests), representation test (RESET test from Ramsey and White's neural network test), goodness-of-fit, the Chow test for stability of prediction, optimal prediction, partial adjustment models, ADL models, LSTAR models and smooth transitions.
- Intervention analysis and transference models, conditional heteroscedasticity models (ARCH, GARCH, IGARCH, GARCH-M and EGARCH).

Algebra in Data Science (4.8 ECTS)

Hours per semester: 36
Course code: CM0888
Software/language used: python and Matlab
Summary:

(This course is part of the Master in Data Science and Analytics at EAFIT) This course is an introduction to linear algebra and mathematics for basic data science and analytics. In particular, this course covered the following topics:

- Basics from set theory, inferior and superior limits of a sequence of sets.
- Metric spaces and properties, examples of metrics, interpretations and graphic representation of metrics, open and closed balls, neighborhoods, the Mahalanobis distance.
- Definition of normed vector spaces and properties, examples of norms, interpretation of norms in data analysis and covariances.
- Definition of spaces with inner product and properties, examples of inner products and the concept of covariance.
- General concepts of matrices from Linear Algebra, operations, types and properties, determinants, trace, inverse matrices, orthogonal matrices, covariance and correlation matrices, generalized dispersion metrics, matrix norms, the Frobenius norm, Kronecker product, LU, Cholesky and QR matrix decomposition.
- Hilbert matrix, matrix conditioning, pseudo-inverse, the Ledoit-Wolf covariance estimation matrix for robustness and conditioning, Minimum Covariance Determinant and Oracle Approximating Shrinkage estimator.
- Principal component analysis, singular value decomposition, spectral decomposition of matrices, eigenvalues and eigenvectors, application of spectral decomposition for image analysis.