UNIVERSAL TEICHMÜLLER SPACE AND IT QUANTIZATION

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Universal Teichmüller space $\mathcal{T}$ is the quotient of the group $\mathrm{QS}(S^1)$ of quasisymmetric homeomorphisms of the unit circle S^1 (i.e. homeomorphisms of S^1 extending to quasiconformal homeomorphisms of the unit disc) modulo Möbius transformations. It contains the quotient $\mathcal{S}$ of the group $\mathrm{Diff}_+(S^1)$ of diffeomorphisms of S^1 modulo Möbius transformations. Both groups act naturally on the Sobolev space $H := H_0^{1/2}(S^1, \mathbb{R})$ of half-differentiable functions on S^1.

Quantization problem for $\mathcal{T}$ and $\mathcal{S}$ arises in string theory where these spaces are considered as phase manifolds. To solve the problem for a given phase space means to fix a Lie algebra of functions (observables) on it and construct its irreducible representation in a Hilbert (quantization) space.

For the space $\mathcal{S}$ of diffeomorphisms of S^1 the algebra of observables coincides with the Lie algebra $\mathrm{Vect}(S^1)$ of $\mathrm{Diff}_+(S^1)$. Its quantization space is identified with the Fock space $F(H)$, associated with the Sobolev space H. Infinitesimal version of the $\mathrm{Diff}_+(S^1)$-action on H generates an irreducible representation of $\mathrm{Vect}(S^1)$ in $F(H)$, yielding a quantization of $\mathcal{S}$.

For the universal Teichmüller space $\mathcal{T}$ the situation is more subtle since $\mathrm{QS}(S^1)$-action on $\mathcal{T}$ is not smooth. Respectively, there is no classical Lie algebra, associated to $\mathrm{QS}(S^1)$. However, we can define a quantum Lie algebra of observables $\mathrm{Der}^q(\mathrm{QS})$, generated by quantum differentials, acting on $F(H)$. These differentials arise from integral operators $d^q h$ on H with kernels, given essentially by finite-difference derivatives of $h \in \mathrm{QS}(S^1)$.

We do not assume any preliminary knowledge from the quantization theory or theory of quasiconformal maps.

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