Bent and generalized bent function into the cyclic group $\mathbb{Z}_{2^k}$

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For two finite abelian groups A, B we call a function $f : A \to B$ bent if $|\sum_{x\in A} \chi(x, f(x))| = \sqrt{|A|}$ for all characters χ of $A \times B$ which are nontrivial on $\{0\} \times B$, or equivalently if the graph $R = \{(x, f(x)) : x \in A\}$ of f is a relative difference set in $A \times B$ relative to B. The standard examples are bent functions from $\mathbb{F}_2^n$ to $\mathbb{F}_2^k$ of which one knows many examples even for the maximal possible value of k, $k = n/2$. If $\mathbb{F}_2^k$ is exchanged with the cyclic group $\mathbb{Z}_{2^k}$, only a few examples are known, the standard examples are relative difference sets obtained from spreads.

Generalized bent functions from $\mathbb{F}_2^n$ to $\mathbb{Z}_2^k$ are defined as functions that satisfy the weaker condition that

$$\sum_{x\in\mathbb{F}_2^n} \epsilon^{f(x)}(-1)^{u\cdot x}, \quad \epsilon = e^{2\pi i/2^k},$$

has absolute value $2^{n/2}$ for all $u \in \mathbb{F}_2^n$, i.e. only the characters of $\mathbb{F}_2^n \times \mathbb{Z}_{2^k}$ of order 2^k are considered. Generalized bent functions were intensively studied in the last few years and are by now quite well understood.

We summarize the achievements on (generalized) bent functions into $\mathbb{Z}_{2^k}$, point to connections with earlier works on Boolean bent functions and discuss some open problems on (generalized) bent functions into $\mathbb{Z}_{2^k}$, respectively relative difference sets in $\mathbb{F}_2^n \times \mathbb{Z}_{2^k}$. Parts of the results presented in this talk are obtained in joint work with Alexander Pott, Otto von Guericke University Magdeburg.