# *Strongly Regular Graphs arising from Non Weakly Regular Bent Functions*

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## Section 1: Bent Functions and Their Duals in Odd Characteristics

- $f : \mathbb{F}_{p^n} \rightarrow \mathbb{F}_{p^m}$
- $\alpha \in \mathbb{F}_{p^n}^*, \quad D_\alpha : \mathbb{F}_{p^n} \rightarrow \mathbb{F}_{p^m}, \quad x \longmapsto f(x+\alpha) - f(x)$
- $f$ is perfect nonlinear if $D_\alpha f$ is balanced for every $\alpha \in \mathbb{F}_{p^n}^*$
- Note if $f$ is linear, then $D_\alpha f$ is constant.

# *Section 1: Bent Functions and Their Duals in Odd Characteristics*

- $f : \mathbb{F}_{p^n} \longrightarrow \mathbb{F}_p$
- $\lambda \in \mathbb{F}_{p^n}$

  $\hat{f} : \mathbb{F}_{p^n} \longrightarrow \mathbb{C}$

  $\lambda \longmapsto \sum_{x \in \mathbb{F}_p^n} \epsilon_p^{f(x) - Tr(\lambda x)}$ whrere $\epsilon_p = e^{\frac{2\pi i}{p}}$.
- $f$ is bent if $| \hat{f}(\lambda) |= p^{\frac{n}{2}}$ for all $\lambda \in \mathbb{F}_{p^n}$
- $f$ is bent $\Leftrightarrow$ $f$ is perfect nonlinear (in this sitiuation).

# Section 1: Bent Functions and Their Duals in Odd Characteristics

- $p \geq 3$ prime,
- $f : \mathbb{F}_{p^n} \to \mathbb{F}_p$ bent.
- $\hat{f}(\lambda) = \xi_\lambda p^{\frac{n}{2}} \epsilon_p^{f^*(\lambda)}$ for all $\lambda \in \mathbb{F}_{p^n}$

  Here $\xi_\lambda \in \mathbb{C}$, $|\ \xi_\lambda\ |= 1$ and $f^* : \mathbb{F}_{p^n} \Rightarrow \mathbb{F}_p$ a function.

  Moreover:

- $n$ : even or [$n$ : odd and $p \equiv 1$ *mod* 4] $\Rightarrow \xi_\lambda \in \{-1, 1\}$.
- [$n$ : odd and $p \equiv 3$ *mod* 4] $\Rightarrow \xi_\lambda \in \{-i, i\}$.

# *Section 1: Bent Functions and Their Duals in Odd Characteristics*

- $p \geq 3$ odd,
- $f : \mathbb{F}_{p^n} \Rightarrow \mathbb{F}_p$ bent.
- $f$ is weakly regular bent if there exist $\lambda \in \mathbb{C}$ such that $\hat{f}(\lambda) = \xi p^{\frac{n}{2}} \epsilon_p^{f^*(\lambda)}$ for all $\lambda \in \mathbb{F}_{p^n}$
  This means $\xi$ is independent from $\lambda$.
- If $f$ is bent but not weakly regular, then $f$ is non weakly regular bent.
- There exist weakly regular bent functions.
- There exist non weakly regular bent functions.

# *Section 1: Bent Functions and Their Duals in Odd Characteristics*

- $p \geq 3$ prime,
- $f : \mathbb{F}_{p^n} \Rightarrow \mathbb{F}_p$ bent. $\hat{f}(\lambda) = \xi_\lambda p^{\frac{n}{2}} \epsilon_p^{f^*(\lambda)}$ for all $\lambda \in \mathbb{F}_{p^n}$
- Here $f^* : \mathbb{F}_{p^n} \Rightarrow \mathbb{F}_p$ is the dual function of $f$.
- **Fact:** $f$ is weakly regular bent with $\xi \Rightarrow f^*$ is weakly regular bent with $\xi^{-1}$

## *Section 1: Bent Functions and Their Duals in Odd Characteristics*

- $p \geq 3$ prime,
- $f : \mathbb{F}_{p^n} \Rightarrow \mathbb{F}_p$ bent.
- $\hat{f}(\lambda) = \xi_\lambda p^{\frac{n}{2}} \epsilon_p^{f^*(\lambda)}$
- **Fact:** There exist non weakly regular bent $f$ with $f^*$ bent.
- Quate from: A. Cesmelioglu, W. Meidl, A. Pott,
  ”On the dual of non weakly regular bent functions and self-dual bent functions”, Advances in Mathematics of Communications, vol. 7, pp. 425 – 440, 2013.
  page 429 Remark 2 :
  ”. . . The existence of non weakly regular bent functions with the dual $f^*$ is weakly regular is an open problem. . . .”.

- Our new result: (Özbudak, P., 2017)
  $p \geq 3$ prime
  $f : \mathbb{F}_{p^n} \Rightarrow \mathbb{F}_p$ non weakly regular.
  $f^*$ : bent $\Rightarrow f^*$ : non weakly regular.
- In fact we proved that if $f^*$ is bent then $f^{**}(x) = f(-x)$ as in the case of weakly regular bent functions
- Hence we solve the quoted open problem.

# *Section 1: Bent Functions and Their Duals in Odd Characteristics*

- Any bent $g : \mathbb{F}_{p^n} \Rightarrow \mathbb{F}_p$ is of two types.
- **Type +** : $\hat{f}(0) = \epsilon p^{\frac{n}{2}} \epsilon_p^{f^*(0)}, \ \epsilon \in \{1, i\}$
- **Type -** : $\hat{f}(0) = \epsilon p^{\frac{n}{2}} \epsilon_p^{f^*(0)}, \ \ \epsilon \in \{-1, -i\}$
- Observe that type of $f$ is completely determined by its value distribution.

*Remark 1*

*Regular bent functions are of **Type +**. Non weakly regular bent functions are can be of both type. If n is even, weakly regular but not regular bent functions are **Type -** otherwise can be of both type.*

# *Section 2: Partial Difference Sets and Strongly Regular Graphs*

*Definition 2 (Partial Difference Sets)*

Let $G$ be a group of order $v$ and $D$ be a subset of $G$ with $k$ elements. Then $D$ is called a $(v, k, \lambda, \mu)$– partial difference set (PDS) in $G$ if the expressions $gh^{-1}$, for $g$ and $h$ in $D$ with $g \neq h$, represent each nonidentity element in $D$ exactly $\lambda$ times and represent each nonidentity element not in $D$ exactly $\mu$ times.

A PDS is called **regular** if $e \notin D$ and $D^{-1} = D$.

*Example 3*

Let $G$ be the additive group of a finite field $\mathbb{F}_q$ where $q$ is an odd prime power and $q \equiv 1 \bmod 4$. Then the set $D$ of all nonzero squares in $\mathbb{F}_q$ forms a regular $(q, (q-1)/2, (q-5)/4, (q-1)/4)$– PDS in $G$.

# *Section 2: Partial Difference Sets and Strongly Regular Graphs*

Observe that the regular condition of PDSs is not restrictive. If $D$ is a PDS with $e \in D$ and $D^{-1} = D$, then $D \setminus \{e\}$ is also a PDS. The following proposition shows that $D^{-1} = D$ is quite common for PDSs.

**Proposition 1**

*If $D$ is a $(v, k, \lambda, \mu)-$ PDS with $\lambda \neq \mu$, then $D^{-1} = D$.*

A PDS with $\lambda = \mu$ is just an ordinary difference set.

# *Section 2: Partial Difference Sets and Strongly Regular Graph*

### *Definition 4 (Strongly Regular Graphs)*

A graph $\Gamma$ with $v$ vertices is said to be a $(v, k, \lambda, \mu)$– strongly regular graph if

1. it is regular of valency $k$, i.e., each vertex is joined to exactly $k$ other vertices;
2. any two adjacent vertices are both joined to exactly $\lambda$ other vertices and two nonadjacent vertices are both joined to exactly $\mu$ other vertices.

### *Definition 5 (Cayley Graph)*

$G$ : a finite abelian group
$D$ : an inverse-closed subset of $G$ ($0 < D$ and $D = -D$)
$E := \{(x, y) | x, y \in G, \ x - y \in D\}$
$(G, E)$ is called a Cayley graph, denoted by $Cay(G, D)$.

$D$ is called the connection set of $(G, E)$.

**Proposition 2 ( MA)**

*A Cayley graph $\Gamma$ , generated by a subset $D$ of the regular automorphism group $G$, is a strongly regular graph if and only if $D$ is a **regular** PDS in $G$.*

A subset $D$ of $G$ is called **trivial** if either $D \cup \{e\}$ or $(G \setminus D)U\{e\}$ is a subgroup of $G$. It is equivalent to say that the Cayley graph generated by $D \setminus \{e\}$ is a union of complete graphs or its complement. Otherwise, $D$ is called **nontrivial.**

**Proposition 3 ( MA)**

*Let $D$ be a regular $(v, k, \lambda, \mu)-$ PDS with $D \neq G \setminus \{e\}$. Then $D$ is nontrivial if and only if $1 \leq \mu \leq k - 1$.*

# Section 3: Connection with Bent Functions

- Let $D_i = \{x : x \in \mathbb{F}^*_{3^{2m}} | f(x) = i\}$

**Theorem 6 (Tan, Pott and Fang)**

*Let $f : \mathbb{F}_{3^{2m}} \to \mathbb{F}_3$ be a weakly regular ternary bent function such that $f(x) = f(-x)$ and $f(0) = 0$. Then the subsets $D_0$,$D_1$ and $D_2$ of $\mathbb{F}_{3^{2m}}$ are partial difference sets with certain parameters.*

- After a short time, Chee,Tan and Zhang generalize this work by using weakly regular $p-$ary bent functions to construct. strongly regular graphs.
- Let us define the following sets:
- $D = \{x : x \in \mathbb{F}^*_{p^{2m}} | f(x) = 0\}$
- $D_N = \{x : x \in \mathbb{F}^*_{p^{2m}} | f(x)$ is non square$\}$
- $D_S = \{x : x \in \mathbb{F}^*_{p^{2m}} | f(x)$ is non zero square$\}$

### Theorem 7 (Chee,Tan and Zhang)

*Let $f : \mathbb{F}_{p^{2m}} \to \mathbb{F}_p$ be a weakly regular bent function such that $f(x) = f(-x)$ and $f(0) = 0$ . If there exists an integer $l$ with $(l-1, p-1) = 1$ such that $f(ax) = a^l f(x)$ for any $a \in \mathbb{F}_p$ and $x \in \mathbb{F}_{p^{2m}}$, then the sets $D, D_N, D_S$ are regular partial difference sets with certain parameters*

- **Remark 4 ( [4]) :** ”By using MAGMA, we know that the sets $D, D_N, D_S$ are not PDS for non weakly regular bent function $f(x) = Tr(w^7 x^{98})$ over $\mathbb{F}_{3^6}$ , where $w$ is a primitive element of $\mathbb{F}_{3^6}$ . This implies that the weakly regular condition is necessary.”
- **Question:** What if we change the sets?

# Section 3: Connection with Bent Functions

- Let $f : \mathbb{F}_{3^n} \to \mathbb{F}_3$ be a non weakly regular bent function such that $f(x) = f(-x)$
- We divide $\mathbb{F}_{3^n}$ into two subsets as follows.

$$B_0 := \{w : w \in \mathbb{F}_p^n \mid f(x) + w.x \text{ is } \textbf{type +}\} \tag{1}$$

$$B_1 := \{w : w \in \mathbb{F}_p^n \mid f(x) + w.x \text{ is } \textbf{type -}\} \tag{2}$$

- Observe that $f(x) = f(-x)$ for all $x \in \mathbb{F}_{3^n}$ implies $\hat{f}(x) = \hat{f}(-x)$ for all $x \in \mathbb{F}_{3^n}$
- Therefore $B_0 = -B_0$ and $B_1 = -B_1$
- By definition $0 \in B_0$ if $f$ is **type +** and $0 \in B_1$ if $f$ is **type -**

## Section 3: Connection with Bent Functions

### Theorem 8 (Özbudak, Pelen)

*Let $f : \mathbb{F}_{3^n} \to \mathbb{F}_3$ be a non weakly regular bent function such that $f(x) = f(-x)$ Then the sets $B_0^*$ and $B_1^*$ are partial difference sets with certain parameters.*

By using magma we have obtained the following sporadic examples corresponding to Theorem 8.

### Example 9

$g_1 : \mathbb{F}_{3^6} \to \mathbb{F}_3$, $g_1(x) = Tr_6(\lambda^7 x^{98})$ is non-weakly regular **(Type–)**. Dual of $g_1$ is not bent and corresponding partial difference sets and strongly regular graphs are non trivial.

- $B_0$ is a $(729, 504, 351, 342)$-PDS in $\mathbb{F}_{3^6}$
- $B_1^*$ is a $(729, 224, 62, 71)$-PDS in $\mathbb{F}_{3^6}$

### Example 10

$g_2 : \mathbb{F}_{3^4} \to \mathbb{F}_3$, $g_2(x) = Tr_4(a_0 x^{22} + x^4)$ is non-weakly regular, where $a_0 \in \{\lambda^{10}, -\lambda^{10}, \lambda^{30}, -\lambda^{30}\}$ **(Type+)**. Dual of $g_2$ is not bent. $B_0$ is an subgroup of $\mathbb{F}_{3^4}$ and so corresponding partial difference sets and strongly regular graphs are trivial.

- $B_0^*$ is a $(81, 26, 25, 0)$-PDS in $\mathbb{F}_{3^4}$
- $B_1$ is a $(81, 54, 27, 54)$-PDS in $\mathbb{F}_{3^4}$

*Example 11*

$g_3 : \mathbb{F}_{3^3} \to \mathbb{F}_3$, $g_3(x) = Tr_3(x^{22} + x^8)$ is non-weakly regular **(Type$+$)**. It is self dual. $B_0$ is an subgroup of $\mathbb{F}_{3^3}$ and so corresponding partial difference sets and strongly regular graphs are trivial.

- $B_0^*$ is a $(27, 8, 7, 0)$-PDS in $\mathbb{F}_{3^3}$
- $B_1$ is a $(27, 18, 9, 18)$-PDS in $\mathbb{F}_{3^3}$

*Example 12*

$g_4 : \mathbb{F}_{3^6} \to \mathbb{F}_3$, $g_4(x) = Tr_6(\lambda x^{20} + \lambda^{41} x^{92})$ is non-weakly regular **(Type–)**.Dual of $g_4$ is bent. Since $B_1$ is an subgroup of $\mathbb{F}_{3^6}$ and so corresponding partial difference sets and strongly regular graphs are trivial.

- $B_0$ is a $(729, 648, 567, 648)$-PDS in $\mathbb{F}_{3^6}$
- $B_1^*$ is a $(729, 80, 79, 0)$-PDS in $\mathbb{F}_{3^6}$

**Example 13**

$g_5 : \mathbb{F}_{3^6} \to \mathbb{F}_3$, $g_5(x) = Tr_6(\lambda^7 x^{14} + (\lambda^{35} x^{70}))$ is non-weakly regular **(Type−)**. Dual of $g_5$ is not bent. Corresponding partial difference sets are non trivial.

- $B_0$ is a $(729, 504, 351, 342)$- regular PDS in $\mathbb{F}_{3^6}$
- $B_1^*$ is a $(729, 224, 62, 71)$- regular PDS in $\mathbb{F}_{3^6}$

**Remark 14**

*Non trivial strongly regular graphs correspond to $g_1$ and $g_5$ are from a unital: projective $9$ – ary $[28, 3]$ code with weights $24, 27$; $VO^-(6, 3)$ affine polar graph. (See [2])*

*Definition 15 (Association Scheme)*

Let $V$ be a finite set of vertices, and let $\{R_0, R_1, \ldots, R_d\}$ be binary relations on $V$ with $R_0 := \{(x, x) : x \in V\}$. The configuration $(V; R_0, R_1, \ldots, R_d)$ is called an association scheme of class $d$ on $V$ if the following holds:

1. $V \times V = R_0 \cup R_1 \cup \cdots \cup R_d$ and $R_i \cap R_j =$ for $i \neq j$.
2. $R_i^t = R_{i'}$ for some $i' \in \{0, 1, \ldots, d\}$, where $R_i^t := \{(x, y) | (y, x) \in R_i\}$. If $i' = i$, we call $R_i$ is symmetric.
3. For $i, j, k \in \{0, 1, \ldots, d\}$ and for any pair $(x, y) \in R_k$ , the number $\#\{z \in V | (x, z) \in R_i,\ (z, y) \in Rj\}$ is a constant, which is denoted by $p_{ij}^k$.

An association scheme is said to be symmetric if every $R_i$ is symmetric.

**Definition 16 (Translation Scheme)**

$\Gamma_i := (G, E_i),\ 1 \leq i \leq d$ : Cayley graphs on an abelian group $G$.
$D_i$ : connection sets of $(G, E_i)$
$D_0 := \{0\}$.
$(G, \{D_i\}_{i=0}^{d})$ is called a translation scheme if $(G, \{\Gamma_i\}_{i=0}^{d})$ is an association scheme.

Given a $d-$ class translation scheme $(X, \{R_i\}_{i=0}^{d})$, we can take union of classes to form graphs with larger edge sets which is called a fusion.

**Remark 17 (Fusion Scheme)**

*Note that if the fusion gives a translation scheme again, it is called **fusion scheme**. However, it is not the case everytime.*

# Section 4: Cylotomic Schemes and Their Fusions

$\mathbb{F}_q$ : the finite field of order $q$

$\mathbb{F}_q^*$ : the multiplicative group of $\mathbb{F}_q$

$S$ : be a subgroup of $\mathbb{F}_q^*$ s.t. $S = -S$

*Definition 18 (Cylotomic Scheme)*

The partition $\mathbb{F}_q^* \setminus S$ of $\mathbb{F}_q^*$ gives a translation scheme on $(\mathbb{F}_q, +)$, called a cyclotomic scheme.

Each coset (called a cyclotomic coset) of $\mathbb{F}_q^* \setminus S$ is expressed as

$$S_i^{(N,q)} = \gamma^i \langle \gamma^N \rangle, \quad 0 \le i \le N-1,$$

where $N | q-1$ is a positive integer and $\gamma$ is a fixed primitive element of $\mathbb{F}_q^*$

# Section 4: Cylotomic Schemes and Their Fusions

- Let $q = p^n$. The functions $\Psi_a : \mathbb{F}_q \mapsto \mathbb{C}^*, \ a \in \ F_q$, defined by

$$\Psi_a(x) = \epsilon_p^{Tr_{q\backslash p}(ax)}$$

  are all additive characters of $\mathbb{F}_q$.
- Note that $\Psi_a(x) = \Psi_1(ax)$ and $\overline{\Psi_a(x)} = \Psi_a(-x)$.
- Eigenvalues of the cylotomic schemes are given by $\Psi_a(S_i^{(N,q)}), \ \Psi_a \in \hat{G}$ (Group of additive characters), called Gauss periods.
- Observe that , $\Psi_a(S_i^{(N,q)}) = \Psi_1(S_{i+t}^{(N,q)})$ for any $a \in S_t^{(N,q)}$.

- Write $\eta_i = \Psi_1(S_i^{(N,q)})$. Then, the first eigenmatrix of the cyclotomic scheme is given by

$$\begin{pmatrix} 1 & \frac{q-1}{n} & \frac{q-1}{n} & \frac{q-1}{n} & \cdots & \frac{q-1}{n} \\ 1 & \eta_0 & \eta_1 & \eta_2 & \cdots & \eta_{N-1} \\ 1 & \eta_2 & \eta_3 & \eta_4 & \cdots & \eta_0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & \eta_{N-1} & \eta_0 & \eta_1 & \cdots & \eta_{N-2} \end{pmatrix}$$

- It is an interesting problem to determine fusion schemes of a $N$–class cyclotomic scheme on $\mathbb{F}_q$,
- Let $X_j,\ \ j = 1, 2, \ldots, d$ : a partition of $\mathbb{Z}_N$
- The Bannai-Muzychuk criterion (See [1, 10]) implies that $\bigcup_{i \in X_j} S_i^{(N,q)}$ forms a translation scheme iff $\exists$ a partition $Y_h,\ h = 1, 2, \ldots, d$ of $\mathbb{Z}_N$ s.t. each $\Psi_1(\gamma^a \bigcup_{i \in X_j} S_i^{(N,q)})$ is const. according to $a \in Y_h$.

# Section 4: Cylotomic Schemes and Their Fusions

- Let us consider 2–class fusion schemes (strongly regular graphs) of cyclotomic schemes of order $N = \frac{p^n-1}{p-1}$.
- $T_0 := \{log_\gamma x(mod\ N) | Tr_{p^n \setminus p}(x) = 0,\ x \neq 0\}$.
- $X$ : a subset of $\mathbb{Z}_N$
- When is $\Gamma = Cay(\bigcup_{i \in X} S_i^{(N,p^n)})$ strongly regular?
- $\Gamma$ is strongly regular iff $\Psi_1(\gamma^a \bigcup_{i \in X} S_i^{(N,q)}),\ a = 0, 1, \ldots, N-1$, take exactly two values.
- Equivalently it has exactly two distinct eigenvalues.

**Proposition 4 (Delsarte)**

*$Cay(\bigcup_{i \in X} S_i^{(N,p^n)})$ is strongly regular iff $|X \cap (T_0 - a)|$, $a \in \mathbb{Z}_N$, take exactly two values.*

# Section 4: Cylotomic Schemes and Their Fusions

- Let $f : \mathbb{F}_{3^n} \to \mathbb{F}_3$ be a non weakly regular bent function such that $f(x) = f(-x)$
- $q = 3^n, \; N = \frac{3^n-1}{2}$ and $\gamma$ be a primitive element of $\mathbb{F}_{3^n}^*$. For the simplicity let us use the notation $S_i := S_i^{(N,q)}$
- Obviously, $S_0 = \mathbb{F}_3^*$ and $S_a = \{\gamma^a, -\gamma^a\}$ for all $a \in \mathbb{Z}_N$
- Observe that $B_0^* \cup B_1^* = \bigcup_{a \in \mathbb{Z}_N} S_a$
- Let us define the following disjoint sets

$$X_0 := \{a : a \in Z_N \mid S_a \subset B_0^*\}$$

$$X_1 := \{a : a \in Z_N \mid S_a \subset B_1^*\}$$

- Clearly $X_0 \cup X_1 = \mathbb{Z}_N$.

*Lemma 19*

*For all $i \in \{0, 1\}$, $|X_i \cap (T_0 - a)|$, $a \in \mathbb{Z}_N$, take exactly two values.*

*Proof.*

We use character sums such as Gauss and Jacobi sums. Since it has many details we find appropriate to skip it here. □

*Remark 20*

*Indeed, lemma 19 implies, the sets $B_0^*$ and $B_1^*$ are two intersection sets in $PG(n-1, 3)$ which correspond to two weight projective codes.*

*Corollary 21*

*The partition of $\mathbb{F}_{3^n}^*$ with respect to signs of the Walsh transform of a non weakly regular ternary bent function having the property $f(x) = f(-x)$ gives 2−class fusion scheme of the $\frac{3^n-1}{2}$−class cyclotomic scheme on $\mathbb{F}_{3^n}$*

Takk for din interesse . . .

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