Locating arrays with error-correcting ability

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In this talk, we introduce a notion of a locating array with error-correcting ability and give a construction of such arrays.

Let $\mathcal{F} = \{F_1, F_2, \ldots, F_k\}$ be the set of k *factors*. For each factor $F_j \in \mathcal{F}$, let S_j be the set of possible *levels* or *values* of F_j. For a t-subset $\mathcal{K} \subset \mathcal{F}$ of factors, a *t-way interaction* $I_t(\mathcal{K})$ is a t-set of pairs (F_j, σ_j) such that $F_j \in \mathcal{K}$ and $\sigma_j \in S_j$ for each factor $F_j \in \mathcal{K}$. The set of all t-way interactions is written by $\mathcal{I}_t$. Let $\mathcal{I}_{\bar{t}} = \bigcup_{s \le t} \mathcal{I}_s$ and an interaction in $\mathcal{I}_{\bar{t}}$ is called a $\bar{t}$-way interaction.

let $A = (a_{ij})$ be an $N \times k$ array with elements $a_{ij} \in S_j$. Columns of A correspond to k factors in $\mathcal{F}$ and rows correspond to N tests. Each test gives a result 0 (pass) or 1 (fail). Failures are caused by s-way interactions with $s \le t$.

For a t-subset $\mathcal{K} \subset \mathcal{F}$, we say a t-way interaction $I_t(\mathcal{K}) = \big\{(F_j, \sigma_j) : F_j \in \mathcal{K}\big\}$ appears in the row i of A if $a_{ij} = \sigma_j$ for every $F_j \in \mathcal{K}$.

We use $\rho_A(I_t)$ to denote the set of row indices of A in which the t-way interaction I_t appears. Moreover, for a set of interactions $\mathscr{T}$, let $\rho_A(\mathscr{T}) = \bigcup_{I \in \mathscr{T}} \rho_A(I)$. Note that if $\mathscr{T}$ is exactly the set of all interactions causing faults, then the set of tests that fail is exactly $\rho_A(\mathscr{T})$. We wants to determine the set of interactions which causes failures by examining the test results.

Suppose $\mathscr{T}_1$ and $\mathscr{T}_2$ are sets consisting of $\bar{t}$-way interactions with $|\mathscr{T}_1| \le d$, $|\mathscr{T}_2| \le d$. Then, A is called a $(\bar{d}, \bar{t})$-*locating array* (LA) if $\rho_A(\mathscr{T}_1) = \rho_A(\mathscr{T}_2)$ holds if and only if $\mathscr{T}_1 = \mathscr{T}_2$. Moreover, A is said to be a $(\bar{d}, \bar{t})$-LA with *error-correcting ability* e, or simply a $(\bar{d}, \bar{t}, e)$-LA, if $|\rho_A(\mathscr{T}_1) \triangle \rho_A(\mathscr{T}_2)| \ge 2e + 1$ for any sets of $\bar{t}$-way interactions with $|\mathscr{T}_1| \le d$ and $|\mathscr{T}_1| \le d$, where $\triangle$ denotes the symmetric difference. By utilizing a $(\bar{d}, \bar{t}, e)$-LA, we can determine any set $\mathscr{T}$ of $\bar{t}$-way interactions with $|\mathscr{T}| \le d$ even if there are at most e errors among the N test results.

In this talk, we consider a $(\bar{1}, \bar{t}, e)$-LA with q factors and r levels ($|S_j| = r \ge 2$), for $q \equiv 1 \pmod{r}$ being a prime power. Let

$$A_q^{(r)} = (a_{x,y}) \text{ with } x, y \in \mathbb{F}_q,$$

where

$$a_{x,y} = \begin{cases} 0, & \text{if } x = y, \\ i, & \text{if } \chi_r(y - x) = \zeta_r^i \quad (0 \le i \le r - 1). \end{cases}$$

We will show that $A_q^{(r)}$ is a $(\bar{1}, \bar{t}, e)$-LA for some positive integer e when q is large. Especially, it is shown that, for a prime power $q \equiv 3 \pmod{4}$, $A_q^{(2)}$ is a $(\bar{1}, \bar{2}, e)$-LA with $e = \frac{3q - 10\sqrt{q} - 43}{16}$ if and only if $q \ge 11$ and it is a $(\bar{1}, \bar{3}, e)$-LA with $e = \frac{7q - 114\sqrt{q} - 215}{64}$ whenever $q > 293$.

Furthermore, it is shown that we can obtain locating arrays with less number of tests by trancating rows of $A_q^{(r)}$.