Codes for errors of limited magnitude, a short survey

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We consider codes over the *alphabet* $\mathbb{Z}_q$, the integers modulo q.

Let λ and μ be integers, where $0 \leq \mu \leq \lambda$ and $\lambda > 0$.

We consider the channel where $a \in \mathbb{Z}_q$ can be changed to $(a + x) \pmod{q}$ where $-\mu \leq x \leq \lambda$.

The (λ, μ)-*quasi cross* with center $\mathbf{a}$ in $\mathbb{Z}_q^n$ is the set

$$X(\mathbf{a}) = \bigcup_{i=1}^{n} \{\mathbf{a} + x_i\mathbf{e}_i \mid -\mu \leq x_i \leq \lambda\}$$

where $\mathbf{e}_i$ is the i'th unt vector.

A *code* of length n is a set $C \subset \mathbb{Z}_q^n$. The code is a (λ, μ)-*packing* (or single error correcting code) if the quasi-crosses $X(\mathbf{a})$ where $\mathbf{a} \in C$ are disjoint. The code is a (λ, μ)-*covering* if $\bigcup_{\mathbf{a} \in C} X(\mathbf{a}) = \mathbb{Z}_q^n$.

We see that for $q = 2$, a $(1, 0)$-code is an ordinary binary code. More general, a $(q - 1, 0)$ is an ordinary q-ary code. Here we consider $\lambda < q - 1$.

Construction of packing and covering codes for these channels have been studied over the last ten years. In the talk I will describe some of the these codes. They are mainly linear codes for $\lambda \leq 3$.

I will also describe some codes for $n = 2$ and general λ and μ.