Optimum MDS convolutional codes over $GF(2^m)$ and their relation to the trace function

Øyvind Ytrehus

A systematic convolutional encoder of rate $(n-1)/n$ and maximum degree D generates a code of free distance at most $\mathcal{D} = D+2$ and, at best, a column distance profile (CDP) of $[2, 3, \ldots, \mathcal{D}]$. A code is *Maximum Distance Separable* (MDS) if it possesses this CDP. Applied on a communication channel over which packets are transmitted sequentially and which loses (erases) packets randomly, such a code allows the recovery from any pattern of j erasures in the first j n-packet blocks for $j < \mathcal{D}$, with a delay of at most j blocks counting from the first erasure. This paper addresses the problem of finding the largest $\mathcal{D}$ for which a systematic rate $(n-1)/n$ code over $GF(2^m)$ exists, for given n and m. In particular, constructions for rates $(2^m-1)/2^m$ and $(2^{m-1}-1)/2^{m-1}$ are presented which provide optimum values of $\mathcal{D}$ equal to 3 and 4, respectively. A search algorithm is also developed, which produces new codes for $\mathcal{D}$ for field sizes $2^m \leq 2^{14}$. Using a complete search version of the algorithm, the maximum value of $\mathcal{D}$, and codes that achieve it, are determined for all code rates $\geq 1/2$ and every field size $GF(2^m)$ for $m \leq 5$ (and for some rates for $m = 6$).

The main part of the talk is based on joint work with Ángela Barbero.